Boundary Shear Stress and Roughness over Mobile Alluvial Beds

By Peter J. Whiting[1] and William E. Dietrich[2]

Abstract: The resistance to flow in the turbulent rough-flow range depends primarily upon the size, shape, and arrangement of the granular material making up the boundary. We have estimated the hydraulic roughness of mobile alluvial surfaces by inverting sediment-transport formulas to solve for the local boundary shear stress required to predict the observed sediment flux and size. Inserting this shear stress value and a near-bed velocity measurement into the law of the wall yields the roughness scale, z_0, defined as the height above the bed where velocity goes to zero. If the roughness is related to the coarse fraction of the bedload, such as D_{84}, then $z_0 = 0.1D_{84}$. This roughness, obtained from mobile, naturally packed, and heterogeneous-in-size beds is three times greater than that predicted by the Nikuradse formula developed from nearly uniform and smoothly packed surfaces. We detect no variation in roughness with transport stage, implying that the large static and slowly moving grains determine flow resistance and that momentum extraction by saltating grains is minor. Application of this simple roughness algorithm allows convenient and accurate calculation of the local boundary shear stress.

Introduction

For the purposes of examining the mechanisms that control evolution of bed topography and sorting processes, local accurate data on boundary shear stress fields are essential. In particular, the local boundary shear stress responsible for sediment erosion and deposition must be defined. Such data, however, are rare in field studies. Until relatively inexpensive and durable instruments are developed for directly measuring near-bed velocity fluctuations such that the Reynolds stress can be calculated, the local boundary shear stress in the field must be estimated by some indirect method that relies on theoretical or empirical arguments. The most widely used procedures are based on either the assumption that the boundary shear stress can be estimated from the local horizontal component of the pressure gradient force or the assumption that it can be calculated from velocity profile measurements and the law of the wall.

The first assumption requires convective accelerations to be small; in rivers with complex topography this is generally not correct (Dietrich and Whiting 1989). It also requires accounting for resistance due to bedforms such as bars, dunes, and ripples (Einstein and Barbarossa 1952). In the second assumption, velocity measurements are used to define the gradient of velocity above the bed, and the local boundary shear stress, τ_b, is calculated from the law of the wall

[1]Consulting Geologist, EA Northwest Operations, 8577 154th Avenue, NE, Redmont, WA 98052; formerly, Res. Asst., Dept. of Geology and Geophysics, Univ. of California, Berkeley, CA 94720.

[2]Prof., Dept. of Geology and Geophysics, Univ. of California, Berkeley, CA.

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$$\tau_b = \rho U_*^2 = \rho(u_z\kappa)^2\left(\ln\frac{z}{z_0}\right)^{-2} \quad (1)$$

where u_z = the velocity at height z above the boundary; ρ = the fluid density; κ = von Karman's constant equal to 0.40; U_* = the shear velocity; and z_0 = the height above the bed where velocity is projected to go to zero. Considerable error arises with application of this procedure because boundary shear stress is extremely sensitive to the gradient of velocity. Both accurate velocity measurements and exact elevations above the presumed bed level are difficult to obtain in most field studies. Our experience has been that repeated velocity profile measurements with long sampling intervals (many minutes) for individual points yield highly variable estimates of boundary shear stress. This is true even in simple, steady, uniform flow over a fine gravel, where the depth of flow was much greater than the coarsest grain size on the bed, which was static or only weakly mobile. Often these calculated shear stresses were clearly in error (much greater or much smaller than reasonable estimates). In addition, such profiling is so time-consuming as to be impractical in rivers with fluctuating discharge.

A convenient solution to this problem may lie in using a single near-bed velocity measurement in the law of the wall (Eq. 1), employing an argument for boundary roughness as a function of grain size. The advantage of a single near-bed velocity measurement is that it can be done quickly, and by being close to the boundary, it can avoid the strong influences of upstream roughness features such as dunes or bank irregularities. The difficulties of this approach lie in ascertaining that the near-bed boundary flow is indeed logarithmic, and in developing a general roughness argument that uses local grain-size distribution for scaling.

Precedent exists for this approach. Decades of research in laboratory flumes often employed Preston-tube measurements and an assumed or calculated drag coefficient to estimate local boundary shear stress from single measurements (Ippen and Drinker 1962; Hooke 1975), and some work has been accomplished in rivers (Nece and Smith 1970). Average velocity for a vertical in a channel cross section has often been used in the vertically averaged form of the law of the wall (Eq. 1) to relate shear velocity and roughness following Keulegan (1938)

$$U = \frac{U_*}{\kappa}\ln\frac{0.4H}{\dfrac{AD_x}{30}}$$

where U = the vertically averaged flow velocity; H = the depth of flow; A = an empirical coefficient; and D_x = some representative size of the bed material. The roughness height is linked to the size of sediment following Nikuradse (1933).

A number of workers have suggested roughness values (AD_x) from empirical studies: Leopold et al. (1964), Bray (1979), and Hey (1979) suggested $3.5D_{84}$; Limerinos (1970) suggested $2.8D_{84}$; and Gladki (1979) suggested $2.5D_{80}$. The subscripts denote the grain diameter for which 65, 84, or 90% are finer. Most of these empirical data are from reaches of rivers where the effects of dunes, bars, or channel planform add to the total resistance to flow. Prestegaard (1983) has reported that bar resistance alone

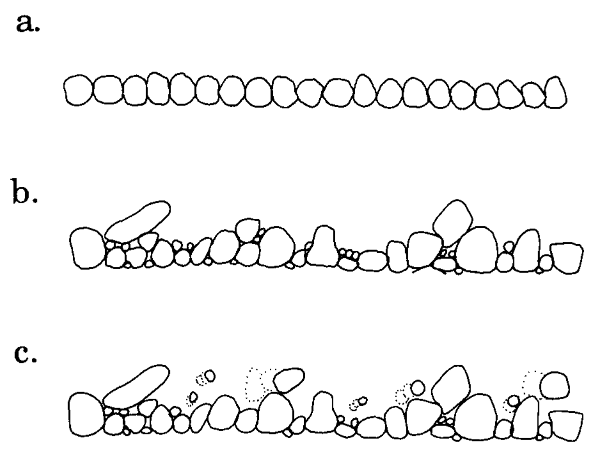

FIG. 1. Surface Texture of: (*a*) Nikuradse's Smoothly Packed and Uniform-Size Grains on Pipe Wall; (*b*) Naturally Packed and Poorly Sorted Grains on Bed; and (*c*) Mobile Bed Like Fig. 1(*b*) Flow Is Left to Right. In Fig. 1(*a*) Thin Coat of Lacquer Covered Grains but Was Ignored for Clarity of Figure

can account for 50% of the total resistance. Parker and Peterson (1980) show that the proportion of the total drag exerted by bars increases as the stage drops.

More recently, Dietrich (1982) proposed using a single-velocity measurement and a theory based upon Smith and McLean (1977) to calculate roughness to map boundary shear stress in rivers. Application in a sand-bedded river bend proved to be quite successful (Dietrich and Smith 1983). The Smith and McLean roughness argument was built upon the earlier work by Owen (1964), hypothesizing that a layer of saltating grains acts to resist the flow by the wakes shed from accelerating grains, and that the magnitude of corresponding roughness is proportional to the thickness of the saltation layer.

Our work in gravel-bedded rivers and our reconsideration of work in the sand-bedded site by Dietrich suggest this roughness analysis needs to be argued differently. Consider Fig. 1, where we sketch the simple surface of uniform grains used by Nikuradse (1933) to relate geometric roughness to hydraulic roughness, a naturally packed heterogeneous-in-size static bed, and its mobile counterpart. The Nikuradse (1933) experiments were for a special case of nearly uniform sediment (0.78 mm–0.82 mm) floated in a layer of lacquer and covered by a thin coat of lacquer such that the variation in grain heights was minimal [Fig. 1(*a*)]. The uniformity of size and the regular arrangement of the grains on the Nikuradse surface minimize its roughness. The Nikuradse relation is commonly applied in the field in spite of the obvious differences between the artificial [Fig. 1(*a*)] and the natural surface [Fig. 1(*b*)]. The natural surface is composed of individual particles of differing size loosely packed in an irregular pattern [Fig. 1(*b*)]. The tops of grains project to different heights above the average bed level because of the variety of pocket geometries in which grains sit, and because of the differing size of the grains. The largest-diameter grains protrude the farthest above the average bed level and consequently expose the greatest surface area to the flow. Alluvial surfaces often exhibit clusters of grains that project substantially above the bed level. Clusters of a few grains, called clast jams (Brayshaw et al. 1983), and broader mobile clusters of many grains, called bedload sheets (Whiting et al. 1988), further roughen the alluvial surface. The momentum of the flow is dissipated in proportion to the magnitude of the squared velocity and the area exposed to flow, hence, these larger grains and clusters should dominate the flow resistance. Now consider the geometry of a poorly sorted bed with grains in motion over the bed (Fig. 1*c*). The irregular surface of various-size grains and clusters of grains projects above the bedload layer defined by the tops of the trajectories of the saltating grains. In addition, the largest grains are moving much slower than the fluid, hence, momentum extraction is like the static case. Thus, we might expect the friction of static and mobile beds to be rather similar.

Here we report analysis of additional data collected from Muddy Creek and data from gravel-bedded Duck Creek, which suggest that for mobile and static beds D_{84} is a useful scale for roughness, as might be inferred from Fig. 1. The analysis is performed by using a procedure similar to Dietrich (1982), but without linking the roughness to the saltation height; we investigate the roughness as it varies with the grain size and transport, inverting sediment transport formula to estimate the local boundary shear stress that must have been acting to transport the observed sediment flux. For the measured near-bed velocity and bedload size, the law of the wall is solved for the roughness scale and this related to the size of sediment.

THEORY

In the region near the bed, sufficiently far from the boundary that the wakes shed from individual grains of roughness are well mixed, and sufficiently close to the boundary that the fluid is unaffected by larger scales of roughness (ripples, dunes, and bar-pool topography), the velocity distribution for steady, uniform flow is logarithmic and the boundary shear stress can be determined from the law of the wall (Eq. 1).

Another way to estimate the local boundary shear stress is to invert sediment-transport formula to solve for the stress required to predict the observed amount of sediment transport. A variety of formulas to predict bedload transport exist that give reasonable results (Meyer-Peter and Muller 1948; Einstein 1950; Yalin 1963; Fernandez Luque and van Beek 1976). One of the most successful formulations was proposed by Meyer-Peter and Muller (1948), modified by Fernandez Luque and van Beek (1976) and can be written as

$$Q_{bl} = \rho_s\left[gD^3\left(\frac{\rho_s - \rho}{\rho}\right)\right]^{0.5} 5.7(\tau_* - \tau_{*cr})^{1.5} \quad (2)$$

where Q_{bl} = the mass rate of bedload transport per unit width; ρ_s = the sediment density (not including porosity); D = the median grain diameter; τ_* = the nondimensional shear stress [$\tau_* = \tau_b/(\rho_s - \rho)gD$]; and τ_{*cr} = the critical nondimensional shear stress for incipient grain motion, which varies between 0.032 and 0.060 for sand and gravel, following Vanoni (1964) and as reproduced in Middleton and Southard (1984). In the case of heterogeneous grain sizes, we will assume that for all the grains on the bed τ_{*cr} is scaled by that for D_{50}.

Combining Eqs. 1 and 2, yields the following equation for the roughness scale (z_0) needed to predict the transport rate given the near-bed flow velocity and the size of material in transport

$$z_0 = ze^{-\beta} \quad (3)$$

where

$$\beta = \frac{u_z\kappa}{\left(\left\{\tau_{*cr} + \left[\frac{Q_{bl}}{5.7\rho_s\left(g\frac{\rho_s - \rho}{\rho}D^3\right)^{0.5}}\right]^{0.67}\right\}gD\frac{\rho_s - \rho}{\rho}\right)^{0.5}} \quad (4)$$

We now have a hydraulic roughness estimate based upon inversion of the Fernandez Luque and van Beek transport formula (1976) and measurement of sediment transport, flow velocity, and sediment size. In order to relate this estimate of hydraulic roughness to the geometry of natural beds, we must select some geometric property to describe the granularity of the bed. Ideally this measure would quantify the differential projection of grains into the flow as influenced by packing, size heterogeneity, and clustering. A logical and practical descriptor of the surface microtopography is the size of the constituent elements. If the basic formulation of Nikuradse (1933) is retained and the roughness is related to some measure of the size of the sediment, we write

$$z_0 = \frac{k_s}{f(\mathsf{R}_*)} \quad (5)$$

Nikuradse related to the size of z_0 to the dimensions of the grain, now called the equivalent sand roughness (k_s), as it varied with the Reynold's roughness number, $\mathsf{R}_* = U_{*cr}k_s/\nu$, where ν is the kinematic viscosity. When $\mathsf{R}_* < 5$, the surface is described as hydraulically smooth and $z_0 = \nu/9U_*$. When $5 < \mathsf{R}_* < 70$, k_s/z_0 is 30–50. And when $\mathsf{R}_* > 70$, the bed is described as hydraulically rough and $k_s/z_0 = 30$.

For uniform sediment, k_s is the size of sediment. For nonuniform sediment, the median is not necessarily the best descriptor of the scale of the roughness. A number of workers have argued that the disproportionate effect of large grains by their projection should be incorporated by characterizing the equivalent sand roughness as some larger-than-median measure of the size distribution. Einstein and El-Samni (1949) suggested D_{65}; Leopold et al. (1964) and Hey (1979) suggested D_{84}; and Limerinos (1970) and Parker and Peterson (1980) suggested D_{90}; where the subscript denotes the grain diameter for which 65, 84, or 90% are finer. For historical and statistical reasons we set k_s proportional to the D_{84} of the sediment

$$k_s = AD_{84} \quad (6)$$

The coefficient (A) is used to incorporate the effects of grain clusters, differential projection of the heterogeneous grains, and nonsystematic packing of the surface. For the nearly uniform Nikuradse surface, A should have a value of 1.0.

Since we have explicitly linked the hydraulic roughness scale to the size of the bed material, it is appropriate to discard the functional dependence on R_*, at least for coarse sand and gravel, and rewrite Eq. 5 as

$$z_0 = \frac{AD_{84}}{30} \quad (7)$$

Consequently the coefficient A additionally incorporates any deviation from a value of 30 (in the denominator of Eq. 7) with Reynolds roughness number.

For the case of a known transport rate, flow velocity, and sediment size, Eqs. 3 and 7 can be combined to estimate the value of the A coefficient

$$A = z \frac{30}{D_{84}} e^{-\beta} \quad (8)$$

Other transport equations can be used similarly to solve for the roughness scale; however, most other formulas require iteration because the shear stress term cannot be isolated.

Field Site and Methods

Several data sets we have collected for the study of bedload transport mechanics are of sufficient detail and accuracy to be useful for the examination of roughness. At two sites, over a sand bed and a gravel bed, and for a broad range of transport stages, we have detailed information on the flow, the material making up the bed surface and bedload, and the transport rate.

Muddy Creek, a clear-flowing, sand-bedded meandering channel in Wyoming, has been studied for more than a decade by Dietrich to understand equilibrium topography in channel bends (Dietrich and Smith 1983, 1984; Dietrich 1987; Dietrich and Whiting 1989). The channel is 5.5-m wide and 0.4-m deep. The bed is covered by three-dimensional dunes, which in the deeper water of the pool have a wavelength of 1–2 m and height of 0.05–0.10 m. In shallower waters of the pointbar top, these features were thinning and lengthening. Over the stoss side of the dunes in the coarser sediment, thin (1–2 grains high) bedload sheets migrate to the dune crest causing short-term fluctuation in transport rate and grain size (Whiting et al. 1988). The collected bedload at Muddy Creek had a median diameter of 0.68 mm and a D_{84} of 1.45 mm. The bedload, bed-surface, and subsurface-size distributions are similar due to the vertical mixing associated with dune passage. Suspended loads are small in this channel. Transport stage (τ_*/τ_{*cr}) values presented here range from 2 to 12. A total of 465 measurements collected over a span of three years are included in this analysis.

Duck Creek is a clear-flowing, gravel-bed irrigation channel in Wyoming. The straight reach studied is 6.0-m wide and 0.5-m deep. Our studies of sediment-transport mechanics led to the identification of bedload sheets (Whiting et al. 1988) and motion-picture quantification of gravel-transport modes and rates (Drake et al. 1988). The study of sediment-transport mechanics included repeated measurement of the near-bed flow velocity and the sediment transport. The median diameter of the bedload collected at Duck Creek was 5.1 mm, whereas D_{84} was 7.7 mm. The bedload and bed-surface size were similar; the average ratio of the median bed-surface size to median bedload size was 1.04. The transport stage was generally less than 2. We used 177 measurements from Duck Creek.

At both sites, a similar methodology was used in the measurement of sediment transport and near-bed flow velocity. A single-impellor current meter suspended from a bridge spanning the channel was positioned near the bed and was constantly observed to maintain its elevation with respect to the bed as erosion or deposition occurred. The meter consists of a 3.5-cm-diameter rotor housed in a 4.2-cm-diameter support ring. Small magnets are embedded in two of the four blades of the rotor; as the rotor spins, a sensor notes the variation in the magnetic field, and the number of revolutions is recorded on a counter. The meters have an accuracy of ±0.3 cm/s or 1.2%, whichever is greater (Smith 1978). Flow velocity was measured for 50–100 sec every 1–2 min for periods of 60–90 min. The open design of the current meter allowed it to be placed near the bed without scour. We held the meter just above the top of the bedload layer, which was equal in both channels to the height of the largest grains rolling on the bed. Occasionally, velocity profiles were made to estimate separately the local boundary shear stress.

Bedload transport was measured with a small sampler held laterally adjacent to the current meter; collection took place simultaneously with velocity measurement. This hand-held sampler is similar in design to a Helley-Smith sampler (Helley and Smith 1971) but has a smaller 2-cm square orifice (Dietrich and Smith 1984). The sampler was continuously observed during collection and thus could be oriented at all times in the direction of local transport. If any scour was observed associated with measurement, the samples were discarded and the measurement repeated. The sampler's efficiency is near 100% based upon comparison with flux determined from both motion pictures (Drake et al. 1988) and from dune migration (Dietrich and Smith 1984). Sample sites at Muddy Creek were concentrated near dune crests and on the flatter portions of the channel in order to minimize near-bed influence of low-momentum fluid shed from the upstream dune. Sample sites at Duck Creek were away from the banks in relatively flat-bed portions of the channel. Periodically, the sampler was used to scoop sediment from the bed surface. Sediment samples were returned to the lab for drying, weighing, and size analysis. The size distribution of the Muddy Creek samples was characterized by settling velocity determination in a 144-cm-long tube (Dietrich 1982). The Duck Creek samples were sieved into quarter-phi fractions for size determination. A total of 642 coupled bedload and velocity measurements from both channels are included in this analysis.

Best-Fit Coefficient of Roughness

The coefficient A minimizing the root mean square error in shear stress prediction $\{[\Sigma(\log \tau_{*pred} - \log \tau_{*'obs'})^2/N]^{0.5}\}$ for different data sets and for three different transport formulas is presented in Table 1. $\tau_{*'obs'}$ is the dimensionless local boundary shear stress that must have been acting to transport the observed flux, and τ_{*pred} is the dimensionless stress from incorporation of the roughness algorithm (Eq. 7) in the law of the wall (Eq. 1). N is the number of measurements. Table 1 also presents the minimum root mean-square error for each case. The three transport equations for which the value of coefficient A was calculated were Fernandez Luque and van Beek (1976), Yalin (1963), and Einstein (1950). A variety of additional bedload equations could have been tried. The commonly used Parket et al. (1982)

TABLE 1. Coefficient A Minimizing Root Mean Square Error in Shear Stress Prediction for Various Data Sets and Transport Formulas

Dataset (1)	N (2)	Einstein formula (3)	Fernandez Luque and van Beek formula (4)	Yalin formula (5)
Muddy Creek	465	4.20 (0.16)	2.94 (0.11)	2.08 (0.09)
Duck Creek	177	1.98 (0.09)	2.96 (0.11)	3.53 (0.10)
Combined	642	2.51 (0.18)	2.95 (0.11)	2.66 (0.11)

Note: Numbers in parentheses are root mean-square error.

equation was not used because of the exclusion of sand in their analysis.

Minimization of root mean-square error in τ_*, as opposed to transport rate, was performed because stress is ultimately what we aim to predict with the roughness algorithm. Moreover, because the Fernandez Luque and van Beek (1976) transport formula contains a critical shear stress term, it is statistically problematic to address transport predictions that are zero because stresses are subcritical.

The range in A values using the variety of transport formulas is moderately large for individual data sets from each channel (2.08–4.20) but similar when data from both channels are combined (2.51–2.95). The Einstein formulation has the broadest range of values for the different data sets (2.08–4.2) and the largest root mean-square error for the combined data sets (0.18). The Yalin formulation has a smaller range of values (2.1–3.5) and a smaller root mean-square error (0.11). The Fernandez Luque and van Beek equation gives consistent A values near 2.95 for both data sets and a mean-square error of 0.11 for the combined set that compares well with Yalin and is much superior to Einstein. Root mean-square errors were larger when we allowed the ratio k_s/z_0 to vary with R_* as in the original Nikuradse formulation than when we kept $k_s/z_0 = 30$. If τ_{*cr} varies between 0.032 and 0.050, as suggested by Yalin (1963), instead of between 0.032 and 0.060, as suggested by Vanoni (1964), A values will be up to 30% smaller for the Duck Creek data near critical boundary shear stresses. The Muddy Creek data is relatively insensitive to this uncertainty because conditions are farther from critical.

Because of the consistent coefficient value of 2.95 for the different data sets using the Fernandez Luque and van Beek equation and the comparable low root mean-square error, we consider this to be the best value for further considerations. The use of this roughness value, the Fernandez-Luque and van Beek transport equation, and the measured near-bed velocity, reveals no systematic scatter in calculated dimensionless shear stress ($\tau_{*\text{pred}}$) and observed dimensionless transport about the Fernandez Luque and van Beek formula (Fig. 2).

The ratio of the shear stress calculated with the roughness algorithm, Eq. 8, to the shear stress required to predict, with the Fernandez Luque and van Beek formula, the observed sediment transport for the measured near-bed velocity and sediment size shows weak structure as a function of D_{84}, flow velocity, dimensionless transport rate $\phi = Q_{bl}/\{\rho_s[gD^3(\rho_s - \rho/\rho)]^{0.5}\}$, and stress from inversion of transport ($\tau_{*\text{'obs'}}$) [Figs. 3(a–d)]. This residual structure is within the variance of the data, hence, it is not examined further.

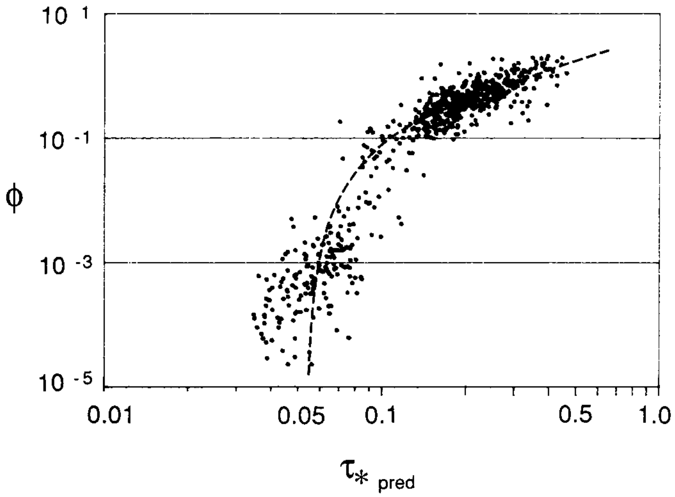

FIG. 2. Fernandez Luque and van Beek Transport Relation (Dashed Line) Compared to Shear Stress Predicted from Eq. 9 and Observed Dimensionless Transport

Apparently the coefficient selected is indistinguishable from a constant value across the range of variables and is not the median value of a systematically varying coefficient.

Within the error of this analysis, the hydraulic roughness from Eq. 7 can be written as $z_0 = 0.1D_{84}$. Rewriting Eq. 1 to incorporate these results, the local boundary shear stress is

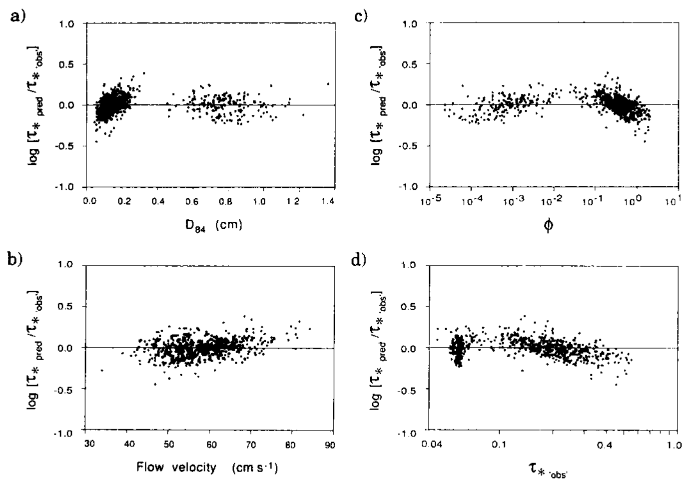

FIG. 3. Residuals in $\tau_{*\text{pred}}/\tau_{*\text{'obs'}}$ versus: (a) D_{84}; (b) Near-Bed Flow Velocity; (c) Dimensionless Transport Rate; and (d) $\tau_{*\text{'obs'}}$

$$\tau_b = \rho(u_z\kappa)^2\left(\ln\frac{10z}{D_{84}}\right)^{-2} \quad (9)$$

Analysis

A number of researchers have reported equivalent sand roughness values (k_s) larger than those of Nikuradse. Ikeda (1983) found $k_s = 1.5D_{84}$ for well-sorted mobile coarse sand in a flume. Hammond et al. (1984) found a roughness value of $k_s = 6.6D_{50}$ for a range of stresses on an offshore gravel bed. They noted only a very slight increase in roughness as the surface became mobile. Van Rijn (1982) analyzed other workers' field and flume lower-plane-bed data and found an equivalent sand roughness near $3D_{90}$. He could not discern an effect of transport stage in the large scatter of roughness values.

Wiberg (personal communication, 1989) recently calculated the roughness of static heterogeneous beds by partitioning the shear stress into components acting on the fluid and on the obstacles in the flow and concluded that the static roughness is approximately $z_0 = 0.11D_{84}$.

The results from Muddy Creek and Duck Creek would seem to corroborate earlier suggestions that the roughness of mobile surfaces may be unaffected by the mobility of grains. We cite as evidence the ratio of local boundary shear stress predicted from the roughness algorithm to the stress required to predict the observed flux [Fig. 3(*d*)]. If mobility of sediment significantly raised the roughness, we expect that use of a single-valued coefficient dependent only upon grain size would strongly underestimate roughness, hence, underestimate the stress at larger dimensionless transport rates, ϕ. This is not what is seen; what we do see is that any systematic deviation from a constant roughness is within the variance of the data.

The constancy of A over four orders of magnitude of dimensionless transport, our observation and calculation of low grain trajectories and concentrations, and our visual observations of static and mobile beds have led us to question the importance of momentum extraction by saltation at least for the majority of lower transport stage flows. Owen (1964), working on eolian saltation, argued that the number of grains in motion was adjusted so that the cumulative extraction of momentum was sufficient to lower the stress at the boundary to the critical value for motion. In contrast, our observations suggest the stochastic nature of sediment transport; 70% of the flux at Duck Creek was associated with sweep events (Drake et al. 1988), presumably when parcels of higher momentum fluid impinged upon the bed. At Duck Creek, we saw the transport as isolated events in both space and time; we could see no local balance between the entrainment and distrainment of grains.

Owen (1969) related the extraction to the thickness of the saltating layer. Because a particle's trajectory depends upon the ratio of shear stress to that particle's critical shear stress for motion, large particles tend to just roll along the bed, whereas smaller grains hop several grain diameters into the flow (Wiberg and Smith 1987). Following Wiberg and Smith, our calculations suggest that the tops of the larger rolling grains are equal or higher than the tops of the trajectory of the smaller grains. Detailed near-bed profiles of sediment transport at Muddy Creek made with stacked arrays of 2-cm aperature samplers (spaced at intervals of 0.5 cm above the bed) show that the amount of sediment collected 0.5 cm above the bed is two orders of magnitude less than collected at the bed. Given that the coarsest particles in motion are rolling along the bed, they are essentially static with respect to the flow. We suggest that although these mobile coarse grains may extract some momentum from the fluid, their effect is equivalent to and largely indistinguishable from the static grains of the bed. At higher τ_* values than reported here momentum extraction apparently is important (Wilson 1989). Interestingly, near the lower bounds of the study of uniform sediment, where the transport stage was 30, Wilson estimated that $k_s \sim 3D_{50}$, a result similar to ours.

Intermediate between the conditions of our study and the study by Wilson (1989) are the conditions used in the analysis of mobile bed roughness by Wiberg and Rubin (1989). They wrote $z_0 = z_{0_s} + z_{0_n}$, where the roughness was comprised of additive terms representing the effect of static grain drag (z_{0_n}) and saltating grain drag (z_{0_s}) [a procedure also used by Dietrich (1982)]. Using the upper-plane-bed data of Guy et al. (1966) ($D_{50} = 0.19$–0.33 mm, transport stage $= 10$–30), they estimated z_0 from projection of the velocity profile to zero velocity, z_{0_n} from the Nikuradse formula, and then solved for z_{0_s}. In contrast to our observations they found that momentum extraction by grains in saltation was significant. As alluded to earlier, the Nikuradse value is appropriate for very smooth beds and may not provide a good estimate of drag over a bed with natural packing. Moreover, the z_0 values from their profiles are very uncertain. For a subset of data when the bed was immobile, the values of z_0 from the profile are 0.011 cm and 0.016 cm values even less than the Nikuradse value of 0.022 cm. It should be noted that because there were typically only a few measurements of velocity within the strict logarithmic layer in Guy's data, Wiberg and Rubin used a formulation for the vertical eddy diffusion coefficient producing a profile thought to be valid through the entire depth, to estimate z_0. Considering the uncertainty in measured z_0 from the velocity profile, and the likely underestimate of z_{0_n}, one could interpret the Wiberg and Rubin results to show no variation in roughness with saltation.

We suggest an alternative view of momentum extraction over heterogeneous mobile beds where the coarse fraction moves largely by rolling and where suspended load is minor. In this case, the momentum is dissipated by the drag of the bed in the same manner as over immobile beds. As the velocity of the fluid increases, the momentum extracted by the bed increases with greater drag on the moving and stationary grains. Finer sediment moves below the tops of the large grains and the tops of clusters, and the coarse fraction is moving sufficiently slow with respect to the fluid that the process of resistance to flow is indistinguishable from the static-bed case.

This view of momentum extraction may have important implications for bedload transport theory. Bagnold (1936, 1973) pointed out the need for a dynamic mechanism to equilibrate the mobile bed. His solution was to propose that the grains in motion extract just the appropriate momentum to lower the shear stress at the boundary to the critical value for entrainment; many have since endorsed this view, including Owen (1964), Smith and McLean (1977), Grant and Madsen (1982), Dietrich (1982), and Wiberg and Rubin (1989). Our data, in combination with recent studies by Drake et al. (1988) pointing to the important role of sweep transport and by Kirchner et al. (1990) defining critical shear stress variability suggest a different solution to this conundrum; as the overall momentum of the flow increases, the static members of the coarser grains experience greater drag and become weakly mobilized. The boundary shear stress is not lowered to some critical value, rather the shear stress remains high and more grains are set in motion.

Application of the Roughness Algorithm

We suggest the following methodology for estimating alluvial roughness and local boundary shear stress based upon our experience in sand and in gravel channels:

1. Estimate the D_{84} of the bed surface.
2. Measure near-bed flow velocity at a known height close to the bed surface.
3. Use Eq. 9 to calculate the local boundary shear stress.

The D_{84} of the bed surface is preferably determined by sieving (or settling tube analysis) a scoop sample of the surface, or by point counting surface grains. A visual estimate may suffice, but will degrade results by its inaccuracy as shown in Fig. 4; for instance, for a reasonable velocity of 50 cm/s at a level 2 cm above a fine gravel bed, the stress will be known to within 25%, if the grain size is known only to a factor of 1.5. This plot also suggests the sensitivity to A values; if A is incorrect by 50%, the calculated shear stress will be in error by 25%. We note that to calculate A values, we used the size of the bedload; this was appropriate since at Duck Creek and Muddy Creek the bed surface and bedload were similar. In many channels, surface armouring makes the bed surface coarser than the bedload. Over a clearly armoured surface at Rio Grande de los Ranchos, New Mexico (Dietrich and Whiting 1989), we used the bed-surface size and flow velocity 5 cm above the gravel bed to calculate the local boundary shear stress with Eq. 9. The average value of 187 dynes cm^{-2} was in close agreement with the independently estimated value of 205 dynes cm^{-2}. This second value

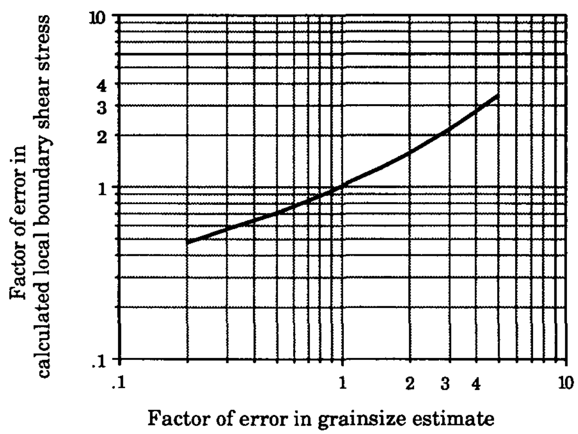

FIG. 4. Sensitivity of Stress Predictions to Error in Grain-Size Estimation and/or A Values

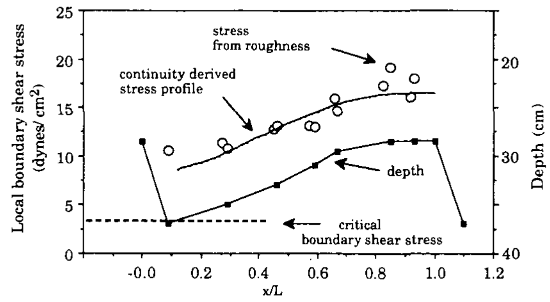

FIG. 5. Local Boundary Shear Stress over Dune at Muddy Creek Predicted Using Eq. 9. Stress Profile from Assumption of Sediment Continuity and Steady Constant-Form Migration for Comparison. X/L Is Distance from Upstream Dune Crest Relative to Dune Wavelength

was estimated by isolating, from the total boundary shear stress following Nelson and Smith (1989), the local boundary shear stress actually exerted on the bed.

Near-bed velocity should be measured with an accurate meter and for sufficient time to average short-term fluctuations. Our experience in sand and gravel channels suggests use of sampling intervals of at least 50–100 sec. Measurements should be made below a height that is two-tenths the flow depth in the region where the law of the wall holds, and sufficiently far from the boundary that wakes of individual particles have coalesced to give a consistent profile. We have found that over coarse sand and fine gravel, velocity measurements near 2 cm above the bed give a good result. If it is possible to measure carefully the sediment transport along with near-bed flow velocity, we suggest recalibration of the roughness algorithm to the particular texture of the studied surface.

Among the variety of techniques for estimating local boundary shear stress (Dietrich and Whiting 1989), the approach presented here has several advantages. As an example, in Fig. 5, we have plotted the variation in stress over a dune at Muddy Creek calculated with the roughness algorithm (Eq. 9). For comparison, we have also plotted the stress profile required by sediment continuity [$\partial Q_{bl}/\partial s = (1 - p)\rho_s \partial z/\partial t$] assuming steady and constant form migration so that the transport rate at the dune crest $Q_{bl} = (1 - p)\rho_s h U_{crest}$, where h is the dune-crest height and U_{crest} is the dune-crest migration rate. Since the suspended load is minor in this channel the conservation of bedload should accurately describe the migration rate of the dune. For calculating the shear stress from the dune migration, we assume the measured average D_{50} of 0.70 mm does not vary over the dune. The roughness algorithm and the law of the wall predict a pattern of increasing local boundary shear stress that is corroborated by the continuity-based stress prediction. An alternative estimate of the shear stress from the slope of the velocity gradient using the law of the wall does not work particularly well because in order to have sufficient points to constrain the profile for the typical size of current meters, the profile projects well into the flow and includes larger-scale drag (Dietrich and Whiting 1989). This overestimates the stress exerted on the boundary. In addition, unless a number of meters are stacked in a vertical array, which

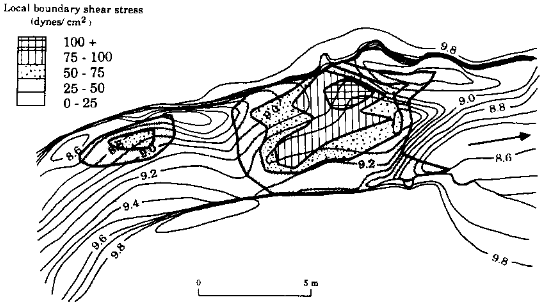

FIG. 6. Local Boundary Shear Stress at Solfatara Creek Estimated from Eq. 9. Bed Elevation Contours in Meters. Water Surface Elevation Was 9.52 m

introduces the problem mentioned previously, the measurement at a number of points in a profile is a time-consuming procedure. The amount of time required to complete a profile becomes important for the case of a migrating dune because migration causes successive measurements at different elevations in a vertical profile to become uncorrelated.

In another example, from Solfatara Creek in Yellowstone National Park, Wyoming (Whiting and Dietrich 1990), we map the local boundary shear stress over a midchannel bar (Fig. 6). Solfatara Creek is a 5.2-m-wide, 0.4-m-deep gravel-bed channel. The median bed-surface grain size is 8.0 mm; and D_{84} is 16.1 mm. During the period studied, discharge was less than one-half bankfull and the bed was largely immobile although the sensitivity of the surface to disturbance suggests shear stresses were near critical. Flow exits an upstream bend and enters a fairly straight 20-m-long reach where shoaling forces sufficiently strong near-bed flow accelerations that there is little variation in velocity for the majority of the profile, even within the expected logarithmic layer near the bed. As a result, velocity profiles and the law of the wall give unreasonably low values of local boundary shear stress, and unreasonably small z_0 values ($k_s/z_0 > 1{,}000$). Alternative methods of calculating the local boundary shear stress such as from the equations of motion or approximated by the pressure gradient force, both require a form drag correction for dunes, bars, and planform irregularities that by its formulation gives an integral, not local, estimate of the boundary shear stress. Moreover, the pressure gradient force could differ greatly from the local boundary shear stress given the flow acceleration. The roughness algorithm coupled with the law of the wall predicts increasing local boundary shear stress to the top of the bar at Solfatara Creek, which suggests the importance of grain-size adjustment in maintaining topography in a diverging stress field that would otherwise lead to planation of the topography (Whiting and Dietrich 1990).

Conclusion

The use of a single near-bed velocity measurement in the law of the wall and an estimate of the surface roughness provides a quick, convenient, and flexible method for calculating local boundary shear stress in hydraulic and geomorphic studies.

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Appendix II. Notation

The following symbols are used in this paper:

A = roughness coefficient;
D = grain diameter; subscript denotes % finer;
g = gravitational acceleration;
H = depth of flow;
h = dune crest height;
k_s = equivalent sand roughness;
L = wavelength of dune;
N = number of values in root mean-square error;
Q_{bl} = bedload transport in g $cm^{-1} \cdot s^{-1}$;
p = porosity, assumed equal to 0.35;
R_* = Reynold's roughness number; $U_{*cr}k_s/\nu$;
S = downstream water surface slope;
s = downstream direction;
t = time;
U = vertically averaged flow velocity;
U_{cr} = dune crest migration velocity;
U_* = shear velocity equal to $(\tau_b/\rho)^{0.5}$;
u_z = flow velocity at height z above bed;
x = distance downstream from dune crest;
z = height above bed;
z_o = height where velocity goes to zero according to the law of the wall (Eq. 1);
κ = von Karman's constant equal to 0.40;
ν = kinematic viscosity;
ρ = fluid density;
ρ_s = density of sediment material (not including pores), assumed equal to 2.65 g cm^{-3};
τ_b = local boundary shear stress;
τ_* = dimensionless local boundary shear stress; $\tau_b/[(\rho_s - \rho)gD_{50}]$; subscripts denote as follows: cr = critical value for entrainment; pred = predicted from roughness algorithm; and 'obs' = required to transport the observed flux; and
ϕ = dimensionless transport rate.